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Unstable Heavy Majorana Neutrinos and Leptogenesis

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Abstract

We propose a new mechanism producing a non–vanishing lepton number asymmetry, based on decays of heavy Majorana neutrinos. If they are produced out of equilibrium, as occurs in preheating scenario, and are superpositions of mass eigenstates rapidly decaying, their decay rates contains interference terms provided the mass differences Δm are small compared to widths Γ. The resulting lepton asymmetry, which is the analogue of the time–integrated CP asymmetry in $B^0 - \overline{B}^0$ system, is found to be proportional to $\Delta m/\Gamma$.

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1 Introduction

A possible mechanism leading to the production, in the early universe, of a baryon–antibaryon asymmetry can be found in terms of lepton number production by heavy Majorana neutrino decays [1]. As well known, the lepton number L_0 so generated is reprocessed by sphaleron transitions, and partially converted into baryon number B

$$B = -\frac{8n_g + 4n_H}{22n_g + 13n_H} L_0 \quad , \tag{1}$$

where n_g is the number of fermion generations, and n_H denotes the number of Higgs doublets of electroweak Standard Model. There are two crucial issues in this mechanism which eventually determine its efficiency in producing the value for the baryon to photon number $\eta \sim (10^{-10} \div 10^{-9})$, fixed by observations on light nuclei abundances.

First of all, the number density of heavy right–handed Majorana neutrinos N_i depends on the mechanism of reheating. Since the produced lepton number is proportional to the number of N_i per comoving volume, it is quite important to have a prediction for this parameter, as function of neutrino masses. This has been considered by many authors in the usual scenario of neutrino production *via* thermal excitations in the bath after reheating. In this case neutrino masses should be smaller than the maximal temperature obtained during the reheating of the universe. It has been recently shown by following in detail the reheating mechanism [2] that a reasonable estimate for this upper bound is of the order of $10^3\ T_{RH}$, where $T_{RH} \simeq \sqrt{\Gamma\ m_{Pl}}$, is the so–called reheating temperature, Γ being the decay rate of the inflaton. The value of T_{RH} is constrained to be in the range $(10^8 \div 10^{10})\, GeV$ in order to avoid an overproduction of gravitinos in SUSY scenarios [3], which would imply that very heavy neutrinos, with masses larger than $10^{13}\ GeV$ would not be significantly produced in the reheating. Even stronger constraints on T_{RH} have been obtained by considering non thermal production of gravitinos [4].

More recently, the neutrino production has been also analyzed in the so–called preheating scenario [5], which corresponds to a resonant particle production during the first inflaton oscillations around the minimum of the potential. The main result of this mechanism is that even neutrinos with masses of the order of $10^{15}\ GeV$ can be efficiently produced in highly non–thermal momentum distribution. As will be clear in the following, this feature of N_i energy spectrum distribution is crucial for our purposes.

The second issue is related to the presence of CP violating contributions to the neutrino decay channels in massless fermions and Higgs bosons, and to the role of interference effects which make these CP violating effects observable. There are two such contributions to the microscopic asymmetry ϵ which have been considered in literature. The first one identified [1] is due to interference of the tree–level amplitudes with the absorptive part of the one loop vertex (ϵ'–like effect)

$$\epsilon_v = \frac{1}{8\pi}\left[\left(hh^\dagger\right)_{11}\right]^{-1}\sum_{j=2}^{n_g}\mathcal{I}m\left[\left(hh^\dagger\right)_{1j}\right]^2 f\left(\frac{m_j^2}{m_1^2}\right) \quad , \tag{2}$$

with m_1 the mass of the lightest Majorana neutrino, $f(x)$ can be found in Ref. [1] and h is the coupling of N_i to massless left–handed leptons and Higgs boson.

More recently, it has been observed that one should also consider contributions coming via interference with one loop self–energy (ϵ–like effect) [6, 7]. For example for two generations one has

$$\epsilon_s = \left[\left(hh^\dagger\right)_{11}\left(hh^\dagger\right)_{22}\right]^{-1}\mathcal{I}m\left[\left(hh^\dagger\right)_{12}\right]^2 \frac{(m_1^2-m_2^2)m_1\Gamma_2}{(m_1^2-m_2^2)^2+m_1^2\Gamma_2^2} \quad . \tag{3}$$

It is worth observing that both asymmetries (2) and (3) very much resemble the CP violation asymmetries for charged B mesons

$$\epsilon_f = \frac{\Gamma(B^+\to f)-\Gamma(B^-\to\bar{f})}{\Gamma(B^+\to f)+\Gamma(B^-\to\bar{f})} \quad , \tag{4}$$

with f denoting an arbitrary final state. As for ϵ_v and ϵ_s, in order to have a not vanishing result one needs more than one contribution to the exclusive decay channel f. In complete analogy to lepton asymmetries, the additional contribution to exclusive $B^\pm$ decay channels is provided by radiative processes, the so–called *penguin* diagrams.

In this paper we consider a different scenario for neutrino decays, which provides already at tree–level in the amplitudes a microscopic contribution to the lepton asymmetry. The mechanism, following again the analogy with B physics, is much reminding the time integrated CP asymmetry in the $B^0-\overline{B}^0$ system [8]. In this case, differently from the charged B, by virtue of $B^0-\overline{B}^0$ oscillations one can produce CP asymmetries already at tree–level.

We consider the case in which heavy Majorana neutrinos are produced out of thermal equilibrium through a preheating mechanism. For arbitrary couplings of these neutrinos to the inflaton field, they are produced as superpositions of the *mass eigenstates* N_i, that we denote as *inflaton eigenstates*, N_α. These states, once produced, propagate as coherent superpositions of the N_i till their eventual decay, if neutrino lifetimes are less than the typical decoherence time due to scatterings in the medium. This constraint leads to conditions on both the couplings h and the Majorana masses, which, however, are neither particularly severe, nor fine tuned.

As in the usual scenario, lepton number is produced via decays in (massless) left–handed fermions ψ_{Lj} (j being the family index) and Higgs bosons Φ, and their C–conjugate particles, right–handed antifermions ψ^c_{Rj} and Φ^c, giving rise to a microscopic asymmetry, for each α

$$\epsilon_\alpha = \sum_{j=1}^{n_g} \frac{\Gamma(N_\alpha \to \psi_{Lj}\Phi) - \Gamma(N_\alpha \to \psi^c_{Rj}\Phi^c)}{\Gamma(N_\alpha \to \psi_{Lj}\Phi) + \Gamma(N_\alpha \to \psi^c_{Rj}\Phi^c)} \quad . \tag{5}$$

Since each N_α is a quantum superposition of the mass eigenstates N_i, their decay amplitudes are linear combinations of $\mathcal{A}(N_i \to \psi_{Lj}\Phi,\ \psi^c_{Rj}\Phi^c)$. Provided the Yukawa matrix explicitly breaks CP invariance, a not vanishing value for ϵ_α can be obtained at tree–level only if these different amplitudes may interfere.

It should be stressed at this point that it is always difficult to deal with unstable particles in the clean framework of Quantum Field Theory, since, in this case, they cannot be identified with asymptotic states in some Hilbert space. Nevertheless, it is obvious to expect that if the Majorana neutrinos would be absolutely stable states, or, in the case of decaying particles, if their mass differences are much greater than their widths Γ_i, any interference would be impossible.

Let us consider an arbitrary final state, like $\psi_{Lj}\Phi$ or $\psi^c_{Rj}\Phi^c$, with invariant mass μ. It can be produced by the decay of neutrinos with mass m_i if μ is, say, in the range $m_i - \Gamma_i/2 \leq \mu \leq m_i + \Gamma_i/2$. If there are two such neutrinos N_i and N_k, whose mass difference is smaller than their *average* width $\Gamma_{ik} = (\Gamma_i + \Gamma_k)/2$, it is impossible to distinguish if this decay product is the result of the decay of N_i or rather N_k. In other words, for the process $N_\alpha \to \psi_{Lj}\Phi$ one expects in this case the contribution of the amplitudes due to both possible processes: $N_\alpha \to N_i \to \psi_{Lj}\Phi$ and $N_\alpha \to N_k \to \psi_{Lj}\Phi$.

The interference between the two amplitudes leads, in general, to not vanishing microscopic asymmetries ϵ_α. To estimate their values we adopt a simple approach, based on the idea that the unstable neutrino can be represented as a superposition of well defined one–particle states with definite mass. The several asymmetries ϵ_α are then found to be proportional to the factors $(m_i - m_j)/\Gamma_{ij}$ and to the imaginary parts of the matrix products $hh^\dagger$. We will also show that they partially cancel each other by virtue of a mechanism analogous to GIM [9].

The paper is organized as follows. In section 2 we briefly summarize the main features of the model describing heavy Majorana neutrino dynamics, whose out of equilibrium production mechanism is discussed in section 3. The estimate of the resulting macroscopic L asymmetry is presented in section 4. Finally, section 5 contains our conclusions and outlooks.

2 The model

Let us consider heavy neutrinos ν_{Ri} and ν^c_{Li} (i denotes the family index) with a Majorana mass term

$$L_M = -\left(\overline{\nu}^c_{Li}\, M_{ij}\, \nu_{Rj} \,+\, \overline{\nu}_{Ri}\, M_{ij}\, \nu^c_{Lj}\right) \quad , \tag{6}$$

with M a symmetric real matrix. This mass term can be diagonalized in terms of a set of Majorana neutrinos N_i, with a latin letter as family index, defined as follows

$$\nu_{Ri} = P_R\, W_{ij}\, N_j \quad , \quad \nu^c_{Li} = P_L\, W_{ij}\, N_j \quad , \tag{7}$$

where W is an orthogonal matrix such that $W^T M W$ is diagonal, and $P_{R,L} \equiv (1 \pm \gamma_5)/2$. Denoting with χ the scalar field behaving, in a certain period of the evolution of the universe, as the inflaton, the production of the heavy neutrinos via the reheating mechanism takes place due to a Yukawa term in the Lagrangian density of the form

$$L_\chi = -\chi\, \left(\overline{\nu}^c_{Li}\, G_{ij}\, \nu_{Rj} \,+\, \overline{\nu}_{Ri}\, G_{ij}\, \nu^c_{Lj}\right) \quad , \tag{8}$$

where again G is a symmetric real matrix. Let us denote with N_α (with a greek letter as index) the basis of G eigenstates. In this basis the Majorana mass matrix M is in general not diagonal, the two basis being connected by an orthogonal transformation

$$N_\alpha = U_{\alpha i}\, N_i \quad . \tag{9}$$

Finally, the heavy neutrinos are also coupled to massless left–handed leptons and to the standard $SU(2)$ Higgs doublet Φ through a Dirac term

$$L_D = -\left(\overline{\psi}^c_{Ri} \cdot \Phi\right) h^*_{ij} N_j - \left(\overline{\psi}_{Li} \cdot \Phi^c\right) h_{ij} N_j \quad , \tag{10}$$

with h the Yukawa coupling matrix in the family space and

$$\psi_{Li} = \begin{pmatrix} \nu_{Li} \\ \\ l^-_{Li} \end{pmatrix} \quad , \qquad \Phi = \begin{pmatrix} \varphi^+ \\ \\ \varphi^0 \end{pmatrix} \quad . \tag{11}$$

3 Neutrino production

In the usual scenario, neutrinos, along with all species of particles with masses below the maximal temperature achieved during reheating, are produced as thermal excitations when the inflaton releases its energy density and the radiation epoch starts. This means that, if the N_i have masses smaller than this temperature, they can be thermally excited. In this case, the N_i field configuration would correspond to a thermal distribution of particles with definite mass given by the eigenvalues m_i of the Majorana mass matrix M. When eventually the temperature decreases down to m_i, if the decays into massless leptons and Higgses take place in a out–of–equilibrium condition, then a macroscopic lepton asymmetry can be produced, provided all Sacharov conditions are satisfied. This scenario has been widely studied, and, as mentioned in the previous section, the effect is basically due to the interference between the tree–level decay amplitudes and the one–loop contributions [1, 6]. In light of the bounds on T_{RH} and the discussion in the Introduction, this scenario seems only viable for Majorana neutrinos with masses lighter than $10^{13}\ GeV$.

We would rather analyze the case when these neutrinos are produced through a preheating mechanism as coherent superpositions of mass eigenstates. In this scenario particles are resonantly excited due to the oscillatory behaviour of the inflaton field $\chi(t) = \chi_0\, cos(m_\chi t)$, m_χ^2 being the second derivative of the inflaton potential at its minimum. In the following we will consider the simplest case of a quadratic potential for χ. It has been shown by several authors, [4], [5], [10]–[14], that is possible to produce spin(0), spin(1/2), as well as spin(3/2) particles through non–perturbative effects. The neutrino field satisfies a Dirac equation on a Friedmann–Robertson–Walker spacetime with an effective time dependent mass matrix $\mathcal{M}$. In conformal time $d\eta = dt/R(t)$, with $R(t)$ the scale factor

$$\left[\frac{i}{R}\gamma^\mu\partial_\mu + i\frac{3}{2R^2}\frac{dR}{d\eta}\gamma^0\right] N_i = \mathcal{M}_{ij}(\eta)N_j \quad , \tag{12}$$

with

$$\mathcal{M}_{ij}(\eta) = M_{ij} + \chi(\eta)\, G_{ij} \quad . \tag{13}$$

Equation (12) describes oscillators with a complex time varying frequency. In the simple case of diagonal M and G matrices the neutrino number density is obtained by using a time dependent Bogoliubov canonical transformation [5]. In the general case, however, since M and G are not necessarily simultaneously diagonal one should first diagonalize via an orthogonal transformation the effective mass $\mathcal{M}$, whose eigenvalues may have quite an involved dependence on $\chi(\eta)$. Let us consider the two possible cases:

i) if the order of magnitude of the matrix elements of M is much larger than the one of $\chi(\eta)\, G$, we may treat the term $\chi(\eta)\, G$ as a perturbation to M. Thus in the Majorana mass eigenbasis, up to the first order in perturbation theory we get

$$\mathcal{M}_{ij}(\eta) \simeq [m_i + \chi(\eta)\, G_{ii}]\ \delta_{ij} \quad . \tag{14}$$

Hence the n_g equations for $i = 1, 2, .., n_g$ are decoupled and can be treated as in Ref. [5]. In this case the inflaton would produce neutrinos already as mass eigenstates, but the resonant condition $\det[\mathcal{M}(\eta)] = 0$ for an explosive production of heavy neutrinos is never satisfied.

ii) in the opposite case, when the time dependent mass term, provided by the coupling to the inflaton, is comparable or even dominant over M, it is convenient to use the *inflaton* basis to write the mass term as

$$\mathcal{M}_{\alpha\beta}(\eta) \simeq [M_{\alpha\beta} + \chi(\eta)\, g_\alpha \delta_{\alpha\beta}] \quad . \tag{15}$$

Thus, the preheating neutrino production occurs for those values of η for which $\det[\mathcal{M}(\eta)]$ vanishes.

In case ii), if we assume, as it will be clear in the following, small off–diagonal terms in the matrix M compared to the diagonal ones, we can write the preheating production condition as

$$\det\left[\mathcal{M}(\eta)\right] = \Pi_{\alpha=1}^{n_g}\left(M_{\alpha\alpha} + \chi(\eta)\, g_\alpha\right) + O\left(\frac{m_{\alpha\beta}}{\overline{m}}\right) = 0 \quad , \tag{16}$$

where $m_{\alpha\beta}$ just denotes the order of magnitude of the off–diagonal mass terms in M and $\overline{m}$ the one of the diagonal entries. At lowest order in the ratio $m_{\alpha\beta}/\overline{m}$, eq. (16) is satisfied if, for some η_*

$$M_{\alpha\alpha} + \chi(\eta_*)\, g_\alpha = 0 \quad . \tag{17}$$

In this case N_α heavy Majorana neutrinos will be resonantly produced. However, since the conditions (16) is only satisfied up to terms of the order $m_{\alpha\beta}/\overline{m}$, this implies that the production rates are suppressed to some extent. In other words it is as these neutrinos at η_* would not be produced as massless but with a mass of the order of $m_{\alpha\beta}^2/\overline{m}$. This effect which leads to an exponential suppression factor of the number of heavy Majorana neutrinos produced [5], can be neglected if the following condition is satisfied

$$\frac{m_{\alpha\beta}^2}{\overline{m}} << \sqrt{g_\alpha\ \chi'(\eta_*)} \quad . \tag{18}$$

Since $g_\alpha \chi'(\eta_*) = g_\alpha m_\chi \chi(\eta_*) \sim \overline{m}\ m_\chi$, the above condition becomes

$$\frac{m_{\alpha\beta}}{\overline{m}} << \left(\frac{m_\chi}{\overline{m}}\right)^{1/4} \quad . \tag{19}$$

The constraint (19) gives for example for the typical values $m_\chi = 10^{13}\ GeV$ and $\overline{m} = 10^{15}\ GeV$, $m_{\alpha\beta}/\overline{m} \leq 10^{-1}$, which does not severely affects the order of magnitude of the off–diagonal elements in $M_{\alpha\beta}$, and still allows for a quite large mixing, given by the non diagonal elements of the matrix U, see eq. (9).

Under the condition (19) one can safely apply the results of Ref. [5] where to solve eq. (12), in case ii), one writes down the momentum expansion for Majorana neutrinos $N_\alpha(\eta, \vec{x})$. By a time dependent Bogoliubov transformation it is then possible to diagonalize the Hamiltonian in terms of quasi–particle creation and annihilation operators and, with a customary procedure, to deduce the number of produced particles as the expectation value of the particle number operator n_α on the quasi–particle vacuum. Depending on the value of the parameter q_α, defined as

$$q_\alpha \equiv \frac{g_\alpha^2\, \chi^2(0)}{4\, m_\chi^2} \quad , \tag{20}$$

with $\chi(0)$ the initial value of the inflaton field configuration, the final number density quite rapidly reaches the bound due to Pauli blocking and can be expressed in terms of the maximal momentum k_{max} of the distribution

$$n_\alpha \simeq k_{max}^3 \simeq m_\chi^2 M_{\alpha\alpha} \quad . \tag{21}$$

At later times, when eventually oscillations are damped to smaller values, the fraction of the inflaton energy density transferred to heavy neutrinos, ρ_α/ρ_χ, is frozen to the value, see Ref. [5],

$$\frac{\rho_\alpha}{\rho_\chi} \simeq \frac{m_\chi^2}{\chi^2(0)}\, q_\alpha \quad . \tag{22}$$

In the framework of a chaotic inflation scenario, from the observed amplitude of the density perturbations on large scales, m_χ is constrained to be of the order of $10^{13}\ GeV$. Furthermore, with $\chi(0) \simeq m_{Pl}$, one gets $\rho_\alpha/\rho_\chi \simeq 10^{-12} q_\alpha$. In the following this result will be used to estimate the final lepton number produced by heavy neutrinos.

4 Neutrino propagation and decay

If neutrinos are produced in the inflaton basis N_α, say at time $t=0$ and with momentum $\vec{k}$, they start evolving as a linear superposition of the mass eigenstates N_j of eigenvalue m_j as follows

$$N_\alpha(x;\vec{k}) = \theta(t) \exp\left\{-iE_j t - \frac{\Gamma_j}{2} t + i\vec{k}\cdot\vec{x}\right\} U_{\alpha j}\, N_j \quad , \tag{23}$$

with $x \equiv (t, \vec{x})$, and $E_j = \sqrt{|\vec{k}|^2 + m_j^2}$. In presence of a medium, scattering processes will tend to destroy the coherence among the components of the wave function $N_\alpha(x;\vec{k})$ before its decay, and to populate the universe with thermalized neutrino mass eigenstates. In order to avoid this, one has to impose the condition

$$\Gamma_\alpha >> n\langle\sigma\rangle \equiv \Gamma_{sc} \quad , \tag{24}$$

where Γ_α denotes the N_α decay rate

$$\Gamma_\alpha = U_{\alpha i} \left(\sum_f \langle f|N_i\rangle\langle N_j|f\rangle \right) U^T_{j\alpha} \quad , \tag{25}$$

where the sum is over all possible final states f. With σ we denote the cross section of the relevant scattering processes, averaged over the incoming particles distribution with number density n. The dominant contribution to Γ_α comes from the two body decay channels $\psi_{Li}\Phi$. If the neutrino masses are of the same order of magnitude $m_i \simeq \overline{m}\ \forall i$, the factor in bracket in eq. (25) simplifies to

$$\sum_f \langle f|N_i\rangle\langle N_j|f\rangle \simeq (hh^\dagger)_{ij}\ \frac{\overline{m}}{8\pi} \Rightarrow \Gamma_\alpha \simeq (hh^\dagger)_{\alpha\alpha}\ \frac{\overline{m}}{8\pi} \quad . \tag{26}$$

Notice that the case of almost mass degenerate neutrinos is actually the scenario we are mostly interested in this paper.

The main contributions to Γ_{sc} correspond to the processes $N_\alpha\,\Phi \longrightarrow N_i\,\Phi$, $N_\alpha\,\psi_{Li} \longrightarrow N_i\,\psi_{Lj}$ and crossed channels, as well as to scattering over the large amount of inflaton quanta χ. Condition (25) constraints more severely the Yukawa couplings h and G when the mean energy of the massless Φ, ψ_{Li} and χ is larger than the neutrino masses. This is due to the rapid increasing with this mean energy of the product $n_{\Phi,\psi_L,\chi}\,\langle\sigma\rangle$. In fact, defining, in the N_α rest frame, the *effective* temperature T_*, which represents the mean energy of the massless fermions or Higgses, and similarly T_χ the one of the inflaton excitations, we have, for scattering over massless fermions and Higgses

$$\langle\sigma(\psi_L,\Phi)\rangle \simeq \frac{1}{8\pi\ \overline{m}\ T_*}\left[(hh^\dagger)_{\alpha\alpha}\sum_{j=1}^{n_g}(hh^\dagger)_{jj} + (hh^\dagger hh^\dagger)_{\alpha\alpha}\right] \quad , \tag{27}$$

and similarly, for scattering processes over χ bosons

$$\langle\sigma(\chi)\rangle \sim \frac{1}{8\pi\ \overline{m}\ T_\chi}\ (G^4)_{\alpha\alpha} \quad . \tag{28}$$

For low momentum neutrinos T_* and T_χ also give the mean energy in the comoving frame. Actually from the discussion of section 3, we see that in our preferred choice for $\overline{m} \geq 10^{13}$ GeV and $m_\chi \sim 10^{13}$ GeV, neutrinos are basically emitted as non–relativistic particles, since $k_{max}/\overline{m} \simeq (m_\chi/\overline{m})^{2/3} \leq 1$, thus we can safely neglect any effect due to the difference between the neutrino rest frame and the comoving frame. Since the number density of incoming particles can be expressed as $n_{\psi_L} \simeq n_\Phi \simeq T_*^3$, $n_\chi \simeq T_\chi^3$, the condition for no decoherence becomes

$$\frac{(hh^\dagger)_{\alpha\alpha}\sum_{j=1}^{n_g}(hh^\dagger)_{jj} + (hh^\dagger hh^\dagger)_{\alpha\alpha}}{(hh^\dagger)_{\alpha\alpha}}T_*^2 + \frac{(G^4)_{\alpha\alpha}}{(hh^\dagger)_{\alpha\alpha}}T_\chi^2 \leq \overline{m}^2 \quad . \tag{29}$$

No particularly fine tuned condition on the Yukawa couplings h and G follows from (29) even for very heavy Majorana neutrinos, $\overline{m} \sim 10^{15}\ GeV$. Assuming in fact the extreme limit $T_\chi \sim (10^{-2} \div 10^{-1})\ m_{Pl}$ which, still compatible with a classical description of spacetime structure, seems to be suggested by an exponential production of χ quanta by the inflaton oscillating configuration [15], we get, as an extremely conservative estimate, that there is no decoherence if we take the Yukawa couplings G, h up to $(10^{-3} \div 10^{-2})$. This implies the *conservative* upper bound for q_α (20)

$$q_\alpha \lesssim \left(10^6 \div 10^8\right) \quad . \tag{30}$$

If condition (29) is satisfied, heavy neutrino states (23) evolve coherently till their decays into massless fermions and Higgses.

To evaluate more carefully the decay rates in these channels we can write the state at fixed three–momentum $\vec{k}$, $N_\alpha(x;\vec{k})$, as a superposition of asymptotic plane waves as follows

$$N_\alpha(x;\vec{k}) = U_{\alpha j} \int \rho_j(\mu)\, u^\mu_{\vec{k}}(x)\ d\mu \quad , \tag{31}$$

where $u^\mu_{\vec{k}}(x)$ is a solution of a Dirac equation with mass μ and momentum $\vec{k}$. The spectral function $\rho_j(\mu)$ realizes the decomposition of the unstable state N_j, with mass m_j and width Γ_j, in plane waves. In the neighbourhood of the resonance value $\mu = m_j$ we can assume that $\rho_j(\mu)$ has a Breit–Wigner behaviour

$$\rho_j(\mu) = \frac{i}{\mu - m_j + i\Gamma_j/2} \quad . \tag{32}$$

The expression (31) is clearly inspired by the spectral *Källen–Lehmann* representation of two–point functions of Heisenberg interacting fields. Of course in the limit $\Gamma_j \to 0$ all spectral functions reduce to δ–functions. We will always consider hereafter the limit of narrow resonances, namely $\Gamma_j << m_j$. However, depending on the Majorana neutrino mass spectrum, it is possible that two or more ρ_j significantly overlap. This occurs whenever $|m_i - m_j| < (\Gamma_i + \Gamma_j)/2$.

Using (31), the total decay rate of N_α into pairs $\psi_{Lp}\Phi$ as well as into the C–conjugated channels $\psi^c_{Rp}\Phi^c$, are given by

$$\sum_{p=1}^{n_g} \Gamma(N_\alpha \to \psi_{Lp}\Phi) = \Xi^{ij}_\alpha\, I_{ij}(|\vec{k}|) \quad , \qquad \sum_{p=1}^{n_g} \Gamma(N_\alpha \to \psi^c_{Rp}\Phi^c) = \left(\Xi^{ij}_\alpha\right)^*\, I_{ij}(|\vec{k}|) \quad , \tag{33}$$

where

$$\Xi_\alpha^{ij} = U_{\alpha i}(hh^\dagger)_{ij}U_{\alpha j} = \left(\Xi_\alpha^{ji}\right)^* \quad , \quad \text{no sum over } i \text{ and } j \quad , \tag{34}$$

$$I_{ij}(|\vec{k}|) = \frac{1}{8\pi}\int_0^\infty \frac{\mu^2\, d\mu}{\sqrt{|\vec{k}|^2+\mu^2}}\, \rho_i(\mu)\, \rho_j^*(\mu) = I_{ji}^*(|\vec{k}|) \quad . \tag{35}$$

In eq. (35) the factor $\mu^2/(8\pi\sqrt{|\vec{k}|^2+\mu^2})$ is the result of integration over the phase space for final massless particles with fixed initial mass μ and momentum $\vec{k}$. A straightforward computation shows that the microscopic asymmetries ϵ_α are given by

$$\epsilon_\alpha(|\vec{k}|) = \frac{2\sum_{i<j} \mathcal{I}m\,[\Xi_\alpha^{ij}]\ \mathcal{I}m\left[I_{ij}(|\vec{k}|)\right]}{2\sum_{i<j} \mathcal{R}e\left[\Xi_\alpha^{ij}\right]\ \mathcal{R}e\left[I_{ji}(|\vec{k}|)\right] + \sum_{i=1}^{n_g} \Xi_\alpha^{ii}\, I_{ii}(|\vec{k}|)} \quad . \tag{36}$$

From eq. (36) we get that, in order to have at tree–level not vanishing microscopic asymmetries it is necessary that:

i) a CP violating term is contained in the Yukawa couplings h;

ii) the integrals I_{ij} of the spectral functions contain a not vanishing imaginary part.

As already stated, this second condition is realized if at least two of the neutrino masses satisfy the condition $|m_j - m_i| \leq (\Gamma_j + \Gamma_i)/2$, otherwise the two spectral functions have no significant overlap. If $m_j > m_i$, the main contribution to the imaginary part of I_{ij} is expected for $m_i < \mu < m_j$. In this interval, since for μ close to the resonance values m_i and m_j we can assume for $\rho_{i,j}$ a Breit–Wigner behaviour, the phase difference of ρ_i and ρ_j is almost π. Furthermore, even if $m_i = m_j$, an imaginary part for I_{ij} is expected if the two widths are sensibly different. Consequently the microscopic asymmetry is a function of the factors $2(m_j - m_i)/(\Gamma_i + \Gamma_j)$ and/or $2(\Gamma_j - \Gamma_i)/(m_i + m_j)$.

It is of course quite involved to give a precise estimate of the ϵ_α in the general case, basically because we do not know the exact form of ρ_j. As an example, we take here the particular simple form

$$\rho_j(\mu) = \begin{cases} i\,(\mu - m_j + i\Gamma_j/2)^{-1} & , \quad m_j - \Gamma_j/2 \leq \mu \leq m_j + \Gamma_j/2 \\ 0 & , \quad \text{elsewhere} \end{cases} \quad . \tag{37}$$

Defining $\Gamma_{ij} \equiv (\Gamma_i + \Gamma_j)/2$, $m_{ij} \equiv (m_i + m_j)/2$, $\delta_{ij} \equiv (m_j - m_i)/(\Gamma_i + \Gamma_j)$ and $\gamma_{ij} \equiv (\Gamma_j - \Gamma_i)/\Gamma_{ij}$, we also assume for simplicity that both δ_{ij}, $\gamma_{ij} < 1$, thus the asymmetry can be obtained as an expansion in powers of these parameters. In the narrow width limit, a simple calculation up to the second order in δ_{ij} and γ_{ij} gives

$$\begin{aligned} I_{ij}(|\vec{k}|) &\simeq \frac{m_{ij}^2}{2\pi\sqrt{|\vec{k}|^2+m_{ij}^2}}\left[1-\left(\frac{2|\vec{k}|^2+m_{ij}^2}{|\vec{k}|^2+m_{ij}^2}\right)\frac{\Gamma_{ij}}{4\,m_{ij}}\gamma_{ij}+\left(\frac{1-2i}{4}\right)\gamma_{ij}^2\right. \\ &+ \left.(2i-1)\,\delta_{ij}+\left(\frac{1-2i}{4}\right)\left(\frac{2|\vec{k}|^2+m_{ij}^2}{|\vec{k}|^2+m_{ij}^2}\right)\frac{\Gamma_{ij}}{m_{ij}}\,\delta_{ij}\,\gamma_{ij}-(3+2i)\,\delta_{ij}^2\right] . \end{aligned} \quad (38)$$

It is interesting to consider the simple case of only two generations, for which the expression of the L microscopic asymmetry is particularly simple and the orthogonal matrix U is just given by

$$U=\begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix} . \quad (39)$$

A simple calculation gives at the lowest order in δ_{12} and γ_{12}

$$\begin{aligned} \epsilon_{1,2} &= \pm 2\;\delta_{12}\,\frac{\sin(2\theta)\mathcal{I}m\left[(hh^\dagger)_{12}\right]}{(hh^\dagger)_{11}+(hh^\dagger)_{22}\mp\sin(2\theta)\mathcal{R}e\left[(hh^\dagger)_{12}\right]} \\ &= \pm 2\;\delta_{12}\;\lambda_{CP}\frac{(hh^\dagger)_{11}+(hh^\dagger)_{22}}{(hh^\dagger)_{11}+(hh^\dagger)_{22}\mp\sin(2\theta)\mathcal{R}e\left[(hh^\dagger)_{12}\right]} , \end{aligned} \quad (40)$$

with

$$\lambda_{CP}=\sin(2\theta)\frac{\mathcal{I}m\left[(hh^\dagger)_{12}\right]}{(hh^\dagger)_{11}+(hh^\dagger)_{22}} , \quad (41)$$

representing the strength of the CP violating effects. Notice that since we are working with Majorana neutrinos, even for two generations a phase in the matrix h cannot washed away by a simple redefinition of fields. Thus $(hh^\dagger)_{12}$ is in general complex.

It is interesting to observe that at lowest order in δ_{12} the asymmetries $\epsilon_{1,2}$ do not depend on $|\vec{k}|$. However, in the preheating mechanism neutrinos are basically produced with low momenta, thus, even more generally, we expect a weak dependence of $\epsilon_{1,2}$ on $|\vec{k}|$.

There are several features of eq. (40) which is worth observing. First of all the two asymmetries tend to cancel each other, simply because the two numerators of ϵ_1 and ϵ_2 are opposite by the orthogonality of the mixing matrix U. This is exactly analogous to GIM mechanism [9]. However, already at first order the sum $\epsilon_1+\epsilon_2$ does not vanish because the denominator in eq. (40) is different for the two neutrinos. Furthermore, the total lepton asymmetry L is given by the microscopic asymmetries weighted by the corresponding neutrino number densities n_i. As long as $n_1\neq n_2$ the value of L is not expected to vanish, though a partial cancellation still takes place. It should also be pointed out that at lower order there is no contribution from the widths difference parameter γ_{12}, which only enters at higher orders.

Finally notice that the value for the asymmetry is obtained at tree–level in decay amplitudes, while both contributions previously considered, ϵ_v and ϵ_s, depend on matrix elements of $(hh^\dagger)^2$ in the numerator, see eq.s (2), (3). This means that the asymmetries (40), and the analogous ones for the three generation case, can be in principle quite large, though there is a certain cancellation among them, since they are not suppressed by higher powers of the Yukawa couplings. Also notice that, differently than eq. (3), $\epsilon_{1,2}$ are only linearly dependent on δ_{12}, so they are less suppressed in the limit of small δ_{12}. Incidentally, in the usual scheme the Yukawa matrix elements are constrained to be quite small $(hh^\dagger)_{ii} \leq m_i/m_{Pl}$ to have an out–of–equilibrium decay. This bound is no more necessary if the neutrino are already produced, as in the preheating scenario, in a non thermal way, and the reheating temperature T_{RH} is low enough to prevent from a subsequent thermal production of N_i.

Using the results of section 3, we can finally estimate the total lepton number n_L, normalized to specific entropy. Since the energy fractions ρ_α remain constant till the inflaton decay into radiation, after the reheating stage one gets

$$\frac{n_L}{s} \simeq \frac{T_{RH}}{\overline{m}\,\rho_\chi} \sum_{\alpha=1}^{n_g} \epsilon_\alpha\, \rho_\alpha \simeq 10^{-17} \left(\frac{T_{RH}}{10^{10} GeV}\right) \left(\frac{10^{15} GeV}{\overline{m}}\right) \sum_{\alpha=1}^{n_g} \epsilon_\alpha\, q_\alpha \quad . \tag{42}$$

Choosing q_α in the interval $q_\alpha \sim (10^6 \div 10^8)$, a reheating temperature $T_{RH} \simeq 10^{10} GeV$ and for heavy right–handed Majorana neutrinos, $\overline{m} \simeq 10^{15}\ GeV$, one gets

$$\sum_{\alpha=1}^{n_g} \epsilon_\alpha \sim \left(10^{-2} \div 1\right) \quad , \tag{43}$$

where the value for the ratio $n_B/s \simeq n_L/s \sim (10^{-11} \div 10^{-10})$ given by primordial nucleosynthesis has been used. Note that the lower bound for $\sum_{\alpha=1}^{n_g} \epsilon_\alpha$ of eq. (43) strongly decreases if one slightly releases the very *conservative* condition for no decoherence (30). Moreover, since very heavy Majorana neutrinos with mass up to $10^{18}\ GeV$ are still compatible with preheating scenario described in Ref. [5], it is interesting to observe that just increasing the value of $\overline{m}$ of one order of magnitude, by virtue of (20), (29) and (42) one reduces the lower bound for $\sum_{\alpha=1}^{n_g} \epsilon_\alpha$ of the same amount.

From the above considerations it follows that the required value for the ratio n_B/s can be easily obtained for a wide range of the involved parameters, without imposing fine tuned conditions. In particular it is worth noticing that no particular mass degeneracy is necessary, and CP violating effects can be of the same order of the ones present in the Standard Model and measured in $K^0 - \overline{K}^0$ system.

5 Conclusions

In this paper we have considered a new scenario for the production of a primordial lepton number, based on decays of oscillating heavy Majorana neutrinos. In the framework of the preheating mechanism for a non–thermal production of massive fermions, we have stressed the possible role in leptogenesis of coherent superpositions of unstable mass eigenstate neutrinos. The mechanism is similar to the way an observable CP asymmetry is produced in the neutral B meson system, due to the $B^0 - \overline{B}^0$ oscillation in time. In fact if the decaying neutrinos are linear superpositions of mass eigenstates N_i, the observability of the CP violation is achieved if the different N_i may interfere.

We have shown that if at least two neutrinos have small mass difference Δm, compared with their corresponding decay widths Γ, a microscopic asymmetry can be obtained at tree level in the Yukawa matrices, coupling these states to massless fermions and Higgses. The microscopic asymmetry is found to be proportional, for small mass differences to the ratio $\Delta m/\Gamma$. On the other hand, if the neutrino states have masses quite well separated, with respect to their decay widths, the interference effects we have described is absent and to get a non–vanishing microscopic asymmetry one should consider interference of the decay amplitudes at tree level with higher loop contributions [1],[5]–[7].

In this scenario, a crucial feature is that neutrinos are produced with a non–thermal distribution, thus the Sacharov out–of–equilibrium condition is implemented from the very beginning. If, as we have considered, neutrinos rapidly decay before any thermalization may occur, it is possible to avoid any wash out of the final lepton number so produced by inverse processes or scattering, provided the maximal temperature achieved during reheating is smaller than the mass of the lightest of the heavy Majorana.

It is finally worth stressing that, even using very conservative bounds on the couplings involved and Majorana masses, the order of magnitude of the microscopic asymmetries results compatible with CP violation effects in the Standard Model. Actually this result holds for choices of Δm which are not particularly fine tuned.

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